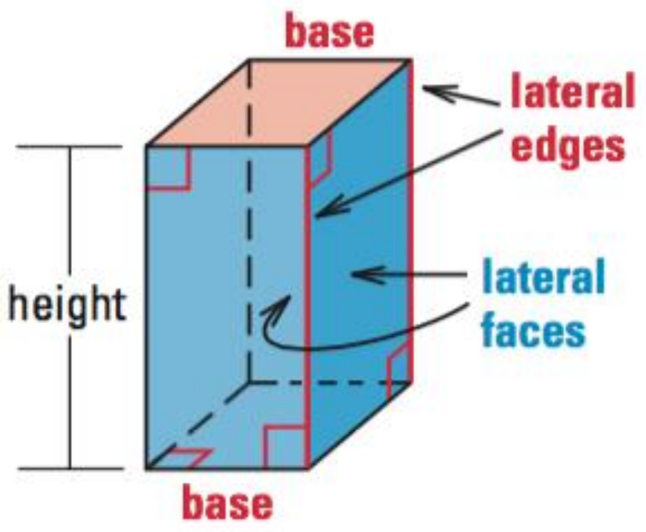
Chapter 12
Surface Area and Volume

Section 2
Surface Area of Prisms and Cylinders

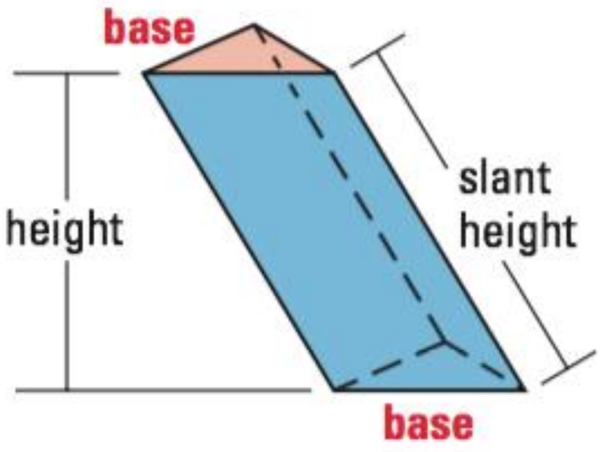
GOAL 1: Finding the Surface Area of a Prism

A **prism** is a polyhedron with two congruent faces, called **bases,** that lie in parallel planes. The other faces, called **lateral faces,** are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are *lateral edges*.

The *altitude* or *height* of a prism is the perpendicular distance between its bases. In a **right prism,** each lateral edge is perpendicular to both bases. Prisms that have lateral edges that are not perpendicular to the bases are **oblique prisms**. The length of the oblique lateral edges is the *slant height* of the prism.



Right rectangular prism

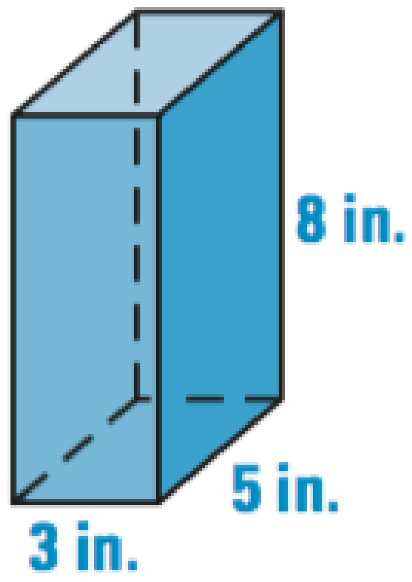


Oblique triangular prism

Prisms are classified by the shapes of their bases. For example, the figures above show one rectangular prism and one triangular prism. The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

Example 1: Finding the Surface Area of a Prism

Find the surface area of a right rectangular prism with a height of 8 inches, length of 3 inches, and a width of 5 inches.



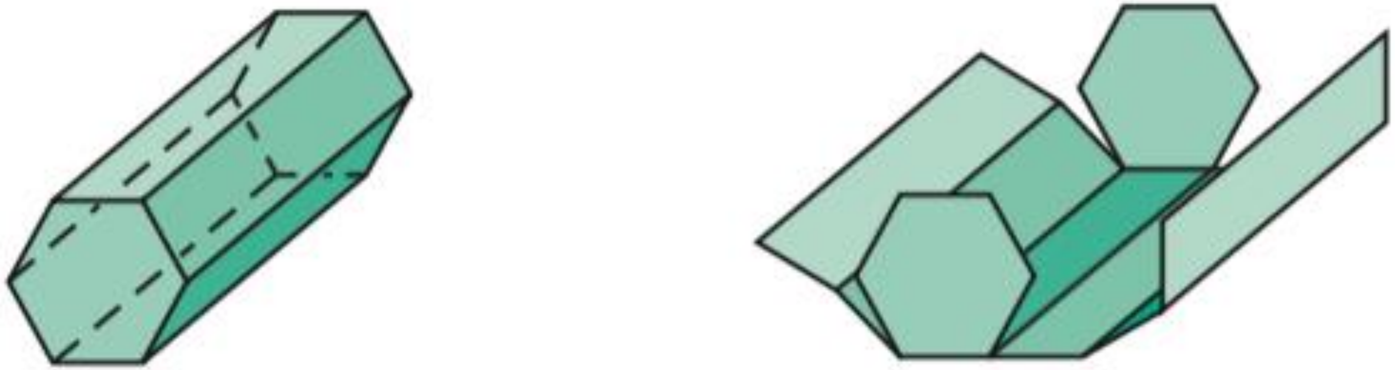
Top/Bottom → 3 x 5 = 15 x 2 = 30

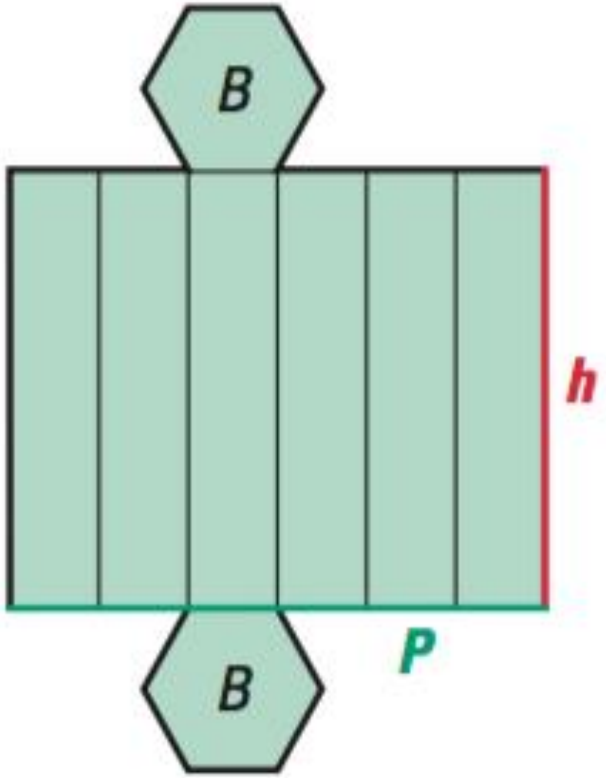
Front/Back → 3 x 8 = 24 x 2 = 48

Left/Right → 5 x 8 = 40 x 2 = 80

Total: 30 + 48 + 80 = 158 inches squared

Imagine that you cut some edges of a right hexagonal prism and unfolded it. The two-dimensional representation of all of the faces is called a **net**.





In the net of the prism, notice that the lateral area (the sum of the areas of the lateral faces) is equal to the perimeter of the base multiplied by the height.

THEOREM

THEOREM 12.2 ***Surface Area of a Right Prism***

The surface area S of a right prism can be found using the formula $S = 2B + Ph$, where B is the area of a base, P is the perimeter of a base, and h is the height.

Rectangular Prism: $2(l \times w) + (2l + 2w)h$

$2lw + 2lh + 2wh$

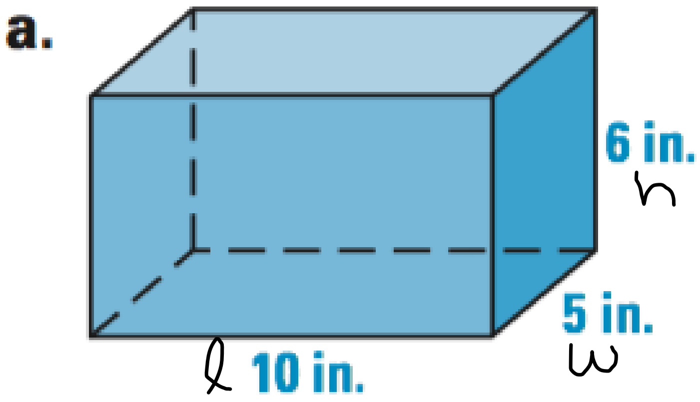
Triangular Prism: $2(\tfrac{1}{2}bh) + (a + b + c)h$

height of base (Δ)

height of prism

Example 2: Using Theorem 12.2

Find the surface area of the right prism.

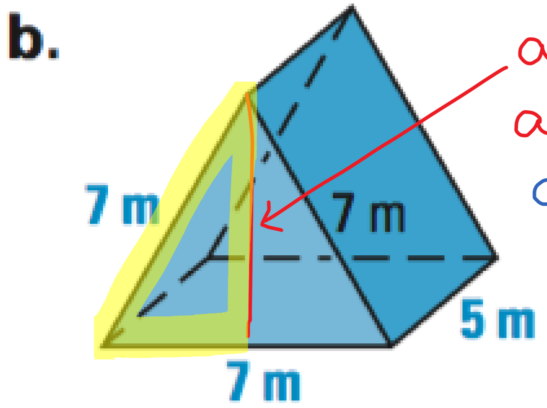


$2lw + 2lh + 2wh$

$2(10)(5) + 2(10)(6) + 2(5)(6)$

$100 + 120 + 60$

280 in^2



$a^2 + b^2 = c^2$

$a^2 + 3.5^2 = 7^2$

$a^2 + 12.25 = 49$

$\sqrt{a^2} = \sqrt{36.75}$

$a = 6.1$

$2(\tfrac{1}{2}bh) + (a+b+c)h$

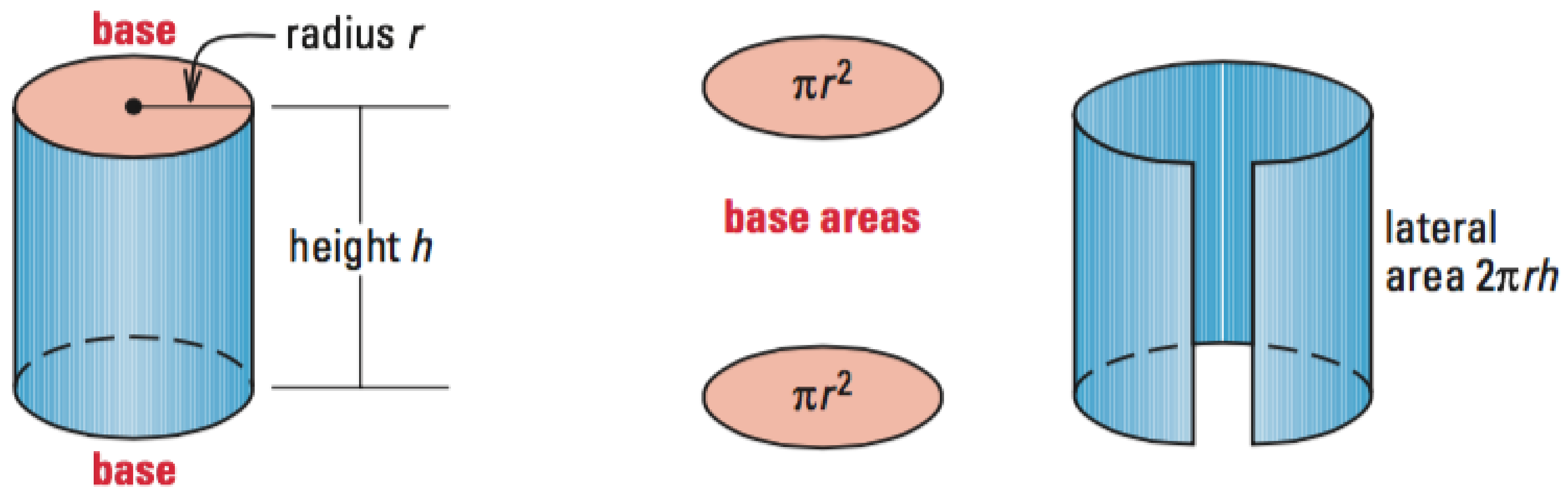
$2(\tfrac{1}{2} \times 7 \times 6.1) + (7+7+7)5$

$42.7 + 105$

147.7 m^2

GOAL 2: Finding the Surface Area of a Cylinder

A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The *altitude*, or *height*, of a cylinder is the perpendicular distance between its bases. The radius of the base is also called the *radius* of the cylinder. A cylinder is called a **right cylinder** if the segment joining the centers of the bases is perpendicular to the bases.



The **lateral area of a cylinder** is the area of its curved surface. The lateral area is equal to the product of the circumference and the height, which is $2\pi rh$. The entire **surface area of a cylinder** is equal to the sum of the lateral area and the areas of the two bases.

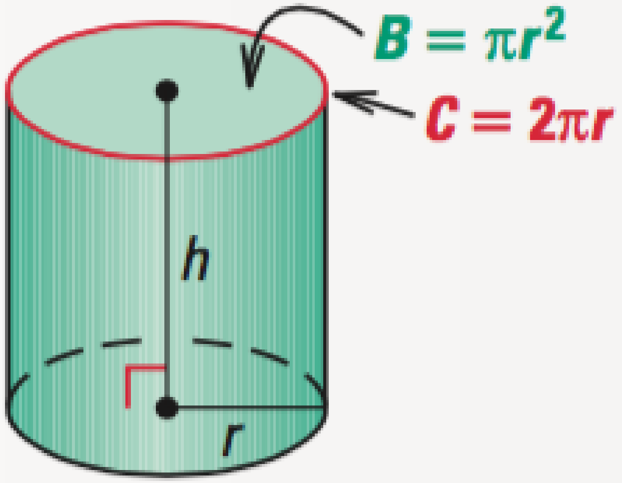
THEOREM

THEOREM 12.3 ***Surface Area of a Right Cylinder***

The surface area S of a right cylinder is

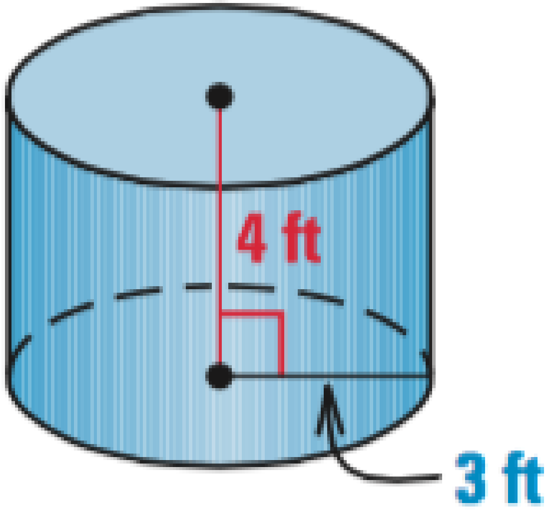
$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



Example 3: Finding the Surface Area of a Cylinder

Find the surface area of the right cylinder.



$2\pi r^2 + 2\pi r h$

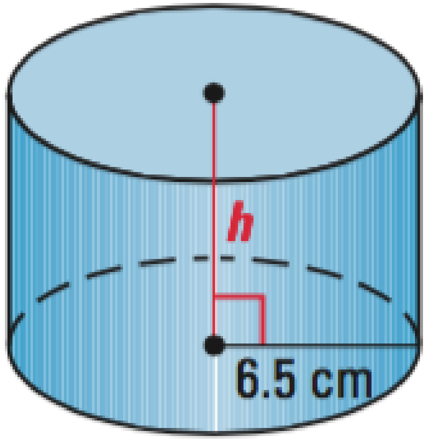
$2(3.14)(3^2) + 2(3.14)(3)(4)$

$56.52 + 75.36$

131.88 ft^2

Example 4: Finding the Height of a Cylinder

Find the height of a cylinder which has a radius of 6.5 centimeters and a surface area of 592.19 square centimeters.



$$A = 2\pi r^2 + 2\pi rh$$

$$592.19 = 2(3.14)(6.5)^2 + 2(3.14)(6.5)h$$

$$592.19 = \cancel{265.33} + 40.82h$$

$$-265.33 \qquad \cancel{-265.33}$$

$$\frac{326.86}{40.82} = \frac{\cancel{40.82}h}{\cancel{40.82}}$$

$$8.01 \text{ cm} = h$$

EXIT SLIP